Variable preconditioning in complex Hilbert space and its application to the nonlinear Schrödinger equation

János Karátson[a], Balázs Kovács[b]

[a] ELTE, Pázmány P. sétány 1/C. 1117 Budapest, Hungary., E-mail: karatson@cs.elte.hu
[b] ELTE, Pázmány P. sétány 1/C. 1117 Budapest, Hungary., E-mail: koboaet@cs.elte.hu

The aim of this talk is to develop an iterative method using stepwise variable preconditioning for the solution of equations $F(u)=b$ in a complex Hilbert space, and to carry out numerical experiments for the nonlinear Schrödinger equation. Thereby we apply time discretization which yields nonlinear elliptic boundary value problems, serving as good examples for our method.

Semilinear parabolic problems arise in many models in meteorology,chemistry and physics, for instance, reaction-diffusion or convection-reaction-diffusion equations, air pollution models. There are also complex problems like Schrödinger type equations.There are many ways to solve such problems numerically. One of the main differences is whether spatial or temporal discretization is applied first, in our case we do the second one (in the first case, for example splitting methods are often used). To be precise, we use the Implicit Euler method, therefore we end up in a nonlinear elliptic boundary value problem on each time step, waiting for its numerical solution, which we do by an iterative, finite element based method. We would like to solve these problems by the thoroughly investigated elliptic iterative theory, see e.g. [2].

The above techniques for nonlinear problems have been usually studied in real Hilbert spaces. Since we would like to apply a method to the complex nonlinear Schrödinger equation, we extend the variable preconditioning method [3] to a complex Hilbert space. This is a kind of quasi-Newton method. In our construction only the real part has to be approximated, and we also need some common norm in which we can compare the contractivity of each step, where it is also natural to use the real part of the analogous norms used in the above mentioned paper [3]. A convergence theorem is given and numerical experiments for the nonlinear Schrödinger equation are presented.

The talk is based on the paper of the authors [1].

[1] Karátson J., Kovács B.: Variable preconditioning in complex Hilbert space and its application to nonlinear Schrödinger equation, submitted to Comput. Math. Appl.
[2] Faragó I., Karátson J.: Numerical Solution of Nonlinear Elliptic Problems via Preconditioning Operators: Theory and Application. Advances in Computation, Volume 11, NOVA Science Publishers, New York, (2002)
[3] Karátson J., Faragó I.: Variable preconditioning via quasi-Newton methods for nonlinear problems in Hilbert space, SIAM J. Numer. Anal. Vol. 41, No. 4, pp. 1242-1262; (2003)