

## Az $\hat{l}^2$ operátor gömbi polárkoordinátákban

$$\hat{l}^2 = -\hbar^2 \left( \frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta} \left( \sin \vartheta \frac{\partial}{\partial \vartheta} \right) + \frac{1}{\sin^2 \vartheta} \frac{\partial^2}{\partial \varphi^2} \right)$$

## A Laplace-operátor gömbi polárkoordinátákban

$$\hat{\Delta} = -\frac{1}{2} \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} (-\hat{l}^2) \right)$$

## A hidrogén atom sajátfüggvényei

$1s$	$\psi_{100} = \frac{1}{\sqrt{\pi}} e^{-r}$
$2s$	$\psi_{200} = \frac{1}{4\sqrt{2}\pi} (2-r) e^{-r/2}$
$2p_0$	$\psi_{210} = \frac{1}{4\sqrt{2}\pi} r e^{-r/2} \cos(\vartheta)$
$2p_{\pm 1}$	$\psi_{21\pm 1} = \frac{1}{8\sqrt{\pi}} r e^{-r/2} \sin(\vartheta) e^{\pm i\varphi}$
$3s$	$\psi_{300} = \frac{2}{81\sqrt{3}\pi} (27 - 18r + 2r^2) e^{-r/3}$
$3p_0$	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}} r (6-r) e^{-r/3} \cos(\vartheta)$
$3p_{\pm 1}$	$\psi_{31\pm 1} = \frac{1}{81\sqrt{\pi}} r (6-r) e^{-r/3} \sin(\vartheta) e^{\pm i\varphi}$
$3d_0$	$\psi_{320} = \frac{1}{81\sqrt{6}\pi} r^2 e^{-r/3} (3 \cos^2(\vartheta) - 1)$
$3d_{\pm 1}$	$\psi_{32\pm 1} = \frac{1}{81\sqrt{\pi}} r^2 e^{-r/3} \sin(\vartheta) \cos(\vartheta) e^{\pm i\varphi}$
$3d_{\pm 2}$	$\psi_{32\pm 2} = \frac{1}{162\sqrt{\pi}} r^2 e^{-r/3} \sin^2(\vartheta) e^{\pm 2i\varphi}$

$$\rho(\nu,T)d\nu=\frac{8\pi\,h\nu^3}{c^3}\frac{d\,\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$$

$$\begin{aligned}\hat{J}_a &= i\hbar \left( \cos \chi \csc \theta \frac{\partial}{\partial \varphi} - \cos \chi \cot \theta \frac{\partial}{\partial \chi} - \sin \chi \frac{\partial}{\partial \theta} \right) \\ \hat{J}_b &= i\hbar \left( -\sin \chi \csc \theta \frac{\partial}{\partial \varphi} + \sin \chi \cot \theta \frac{\partial}{\partial \chi} - \cos \chi \frac{\partial}{\partial \theta} \right) \\ \hat{J}_c &= -i\hbar \frac{\partial}{\partial \chi} \\ \hat{J}_X &= i\hbar \left( \cos \varphi \cot \theta \frac{\partial}{\partial \varphi} - \cos \varphi \csc \theta \frac{\partial}{\partial \chi} + \sin \varphi \frac{\partial}{\partial \theta} \right) \\ \hat{J}_Y &= i\hbar \left( \sin \varphi \cot \theta \frac{\partial}{\partial \varphi} - \sin \varphi \csc \theta \frac{\partial}{\partial \chi} - \cos \varphi \frac{\partial}{\partial \theta} \right) \\ \hat{J}_Z &= -i\hbar \frac{\partial}{\partial \varphi}\end{aligned}$$

$$\hat{J}^2 = -\hbar^2 \left( \csc^2 \theta \frac{\partial^2}{\partial \varphi^2} + \csc^2 \theta \frac{\partial^2}{\partial \chi^2} - 2 \cot \varphi \csc \varphi \frac{\partial^2}{\partial \varphi \partial \chi} + \frac{\partial^2}{\partial \theta^2} + \cot \theta \frac{\partial}{\partial \theta} \right)$$

$$sI_m(s)=k\sum_{i\neq j}A_i(s)A_j(s)r_{ij}^{-1}\cos(\eta_i-\eta_j)\exp(-\ell_{ij}^2s^2/2)\sin[s(r_{ij}-\kappa_{ij}s^2)]$$