Numerical analysis of the Richardson extrapolation in simplified environmental models

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Ordinary or partial differential equations in meteorological problems can only be solved numerically therefore numerical methods are required to give highly accurate results and to be cost-effective.

A frequent problem during numerical modeling is that with a given step size the desired accuracy is not obtained and the inaccurate solution is neglected. In this case the model has to be run again with a smaller step size that requires further computations meaning greater costs.

From this perspective the application of Richardson extrapolation – introduced by meteorologist L. F. Richardson in 1927 – is a good alternative for meteorological modeling purposes.

Two versions of the Richardson extrapolation can be distinguished, which are called passive and active. Both combine two numerical solutions with properly chosen weights, usually one obtained by step size h, providing a result not accurate enough, and one obtained by step size $h/2$. The accuracy of the weighted average of these two solutions is shown to increase from the pth order of the background method to order $p+1$. Therefore a more accurate result is gained by this combination. Moreover, using the aforementioned less appropriate solution obviously makes the method cheaper.

We aim to show that using Richardson extrapolation either combining with operator splitting in a reaction-diffusion problem or with the chosen background methods in a simplified global CO_2 model provides better results than using these schemes alone. It is also shown that using passive Richardson extrapolation is usually a better alternative than using the active version of the method.