Some nonlinear stability notions in numerical analysis

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Many phenomena in nature can be described by mathematical models which consist of functions of a certain number of independent variables and parameters. When we model some real-life phenomenon with a mathematical model, it results in a – usually nonlinear – problem of the form F(u) = 0. The realization of the solving process is very difficult or even impossible. However, we need only a good approximation for the solution of the problem F(u)=0, since our model is already a simplification of the real-life phenomenon.

Therefore we construct numerical models by using some kind of discretization, which results in a sequence of simpler problems, i.e., a numerical method. The notions of consistency and stability are the ones that are independent of the solution of the original problem and are controllable.

Generally, consistency in itself is not enough for convergence. In numerical analysis one of the most important tasks is to guarantee the convergence of the sequence of the numerical solutions. To guarantee this property we introduce the notion of stability.

The primary aim of this talk is to study how to define appropriately the notion of stability in nonlinear problems that helps us to claim that convergence can be replaced by these two notions. This turns out to be useful from the applicational point of view. Sometimes the above described popular "recipe" is summarized as:

„Consistency + Stability --> Convergence"

[1] Faragó, I., Mincsovics, M. E., Fekete, I.: Notes on the basic notions in nonlinear numerical analysis. Electronic Journal of Qualitative Theory of Differential Equations, 1-22 (to appear)